**RESEARCH STUDIES OF STATISTICAL ENERGY METHODS IN SOUND AND STRUCTURAL VIBRATION ANALYSIS**

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# C O N T E N T S

## SUMMARY AND DISCUSSION

During the current reporting period, our studies of statistical energy methods of sound and structural vibration analysis have been centered on three primary phases: (1) radiation resistance of structures, (2) modal densities of structures, and (3) noise transmission. Knowledge of items (1) and (2) is necessary for the successful application of the theory in practical situations, and noise transmission appears to be one of the most promising areas in which the statistical approach can be applied. These activities are congruent with the original objectives of the investigations.

Considerable information has been compiled on the radiation resistance or radiation loss factors for beams, simple panels, orthrotropic plates, and built-up panels. The results of this phase of the study are summarized in Appendix A. Further effort in this area will be directed toward more precise determination of the ranges of applicability of the parameters involved, designing of evaluation experiments, and analytical studies of radiation loss factor for multimodal vibration for shells of revolution.

In the area of noise transmission, a program of study has begun in which the effect of structural shapes on noise reduction will be evaluated. For equal contained volumes, the noise reduction properties of one face (with different geometric shapes) of a cubical will be studied analytically and experimentally in all important frequency ranges. The results thus far of this phase of study are summarized in Appendix B.

The studies of modal density have thus far been concerned with circular cylinders. An experimental arrangement has been designed and set up for further evaluation of predictions that have been reported in the literature. These experiments should be completed in the near future, and the results will be submitted for publication. Appendix C summarizes the approach and progress on modal density studies.

Franklin D. Hart
Project Director

# APPENDIX A

## RADIATION RESISTANCE

Charles J. Runkle

An analysis of the acoustically excited vibration of structures leads to a consideration of the coupling between the sound pressure waves and the induced waves in the structure. In a similar fashion, if one wishes to control the intensity of sound emitted from a vibrating structure, a consideration of the coupling between the vibration of the structure and the induced pressure variations in the surrounding air is necessary.

Coupling can be characterized by a quantity $\mu$,[1*] the resistance ratio, which is a measure of the amount of power radiated from the vibrating body as compared to the total amount of power dissipated both by radiation to the surrounding fluid and by mechanical losses within the structure. Figure 1 illustrates the meaning of $\mu$ with reference to the energy transfer which occurs in the steady-state vibration of a structure.[**]

Since E is the input energy, $\mu$E is the energy dissipated to the surrounding fluid, and $(1-\mu)$E is the energy dissipated within the structure, then

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*Superscript numbers refer to the references listed at the end of the respective appendix.

**All figures in this and succeeding appendices appear at the end of the discussion.

$$\mu = \frac{\mu E}{\mu E + (1-\mu)E}\ .$$

The resistance ratio can be written[1]

$$\mu = \frac{R_{rad}}{R_{rad} + R_{mech}}$$

and since $\eta = (R/\omega_o M)$, then $\mu$ can also be written

$$\mu = \frac{\eta_{rad}}{\eta_{rad} + \eta_{mech}}.$$

The loss factor is related to the damping ratio by $\frac{1}{2}\eta = c_o/c_c$, where the damping ratio is the ratio of the damping coefficient to the critical damping coefficient, which occurs when damping is that specific value necessary to cause the system to be critically damped.

The loss factor, $\eta$, for a plate can be determined by measurements of reverberation time. The reverberation time is the time, T, required for the energy content of the system to decrease by 60 dB; therefore[2] $\eta = 13.8/T_S\omega$. In this case, $\eta$ is the loss factor for the entire system; i.e.,

$$\eta = \eta_{rad} + \eta_{mech}.$$

Then, $\eta_{rad}$ and therefore $\mu$ can be determined by using the equation,[2]

$$S_p/S_a = \omega M \rho T_R \eta_{rad}/27.6\pi^2 n_R c.$$

From the above, it can be seen that an attempt to estimate the degree of coupling between structural and acoustic vibration will require knowledge about either the loss factors or the radiation resistance and the internal resistance. Measurement of $T_R$, $S_a$, and $S_p$ plus information about $n_R$ will give sufficient information to determine the quantities, but in the absence of the opportunity or time for experimental work, theoretical considerations must be available.

The following portion of this report is concerned with the radiation resistance. Results of compiling $R_{rad}$ for various structures and geometrical shapes are presented.

## Beams

Development of mathematical expressions for the determination of the radiation resistance comes about in several ways. Lyon and Maidanik[1] developed an expression by representing the acoustic space and the structure with an equivalent set of oscillators. Such development was based on the following assumptions:

1. The oscillators are linear systems.
2. The response of a single oscillator occurs in a narrow frequency band centered at the natural frequency of the oscillator.
3. Interaction between the oscillators of the acoustic field and the oscillators of the structure occurs only in a narrow frequency band of common natural frequency. Response of the oscillators in all other bands of frequency is negligible.
4. The oscillators are statistically independent so that the total energy of the system is then just the sum

of the individual energies of each oscillator (resonant modes are sufficiently separated in frequency space, have small loss factors, $\eta$, and are linear).

5. The acoustic field is reverberant in the sense that the pressure spectral density can be treated independent of position (i.e., the spectral density is averaged over the volume of the field).

6. The structural vibration field is reverberant in the same manner as the acoustic field.

The general expression for the radiation resistance is[1]

$$R_{rad} = (16/\pi)\rho c k^2 \iint_S \psi(\underline{x}_1, \underline{x}_2)\phi(\underline{x}_1, \underline{x}_2)\, d\underline{x}_1 d\underline{x}_2 \cdots \qquad (1)$$

Evaluation of this expression for an actual case requires that the correlation, $\psi(\underline{x}_1, \underline{x}_2)$ have the same form on the surface of the structure as at points removed from the surface. This simplification restricts the results to structures of small curvature with dimensions which are large compared to the acoustic wavelength. It is further assumed that beam modes are well separated in frequency space.

Lyon and Maidanik[1] applied this formulation to a simply supported beam in an acoustic baffle. Figure 2 shows the beam and its dimensions. The results are as follows:

1. $(k_b/k)<1$ (above coincidence) with $\frac{1}{2}k\ell>1$. This is the range of wavenumbers such that the bending wavelength in the plate is greater than the acoustic wavelength.

a. For $\frac{1}{2}a>1$, where $a=kw[1-(k_b/k)^2]^{1/2}$,

$$R_{rad} \approx S\rho c[1-(k_b/k)^2]^{-1/2}.$$

b. For $\frac{1}{2}a<1$, $R_{rad} \approx \frac{1}{2}S\rho ckw$ .

2. $(k_b/k)>1$ (below coincidence) with $\frac{1}{2}k\ell>1$. In this range of wavenumbers, the bending wavelength in the plate is less than the acoustic wavelength.

a. For $\frac{1}{2}kw>>1$,

$$R_{rad} = S\rho c(k/k_b)^2(1/k\ell)\{[2-(k/k_b)^2]/[1-(k/k_b)^2]^{3/2}\}.$$

b. For $\frac{1}{2}kw<1$,

$$R_{rad} = \rho c(k/k_b)^2(w/\pi\ell)\{1/[1-(k/k_b)^2] + (k_b/2k)\log(k_b+k)/(k_b-k)\}.$$

3. $k_b \approx k$ (in the neighborhood of coincidence) with $\frac{1}{2}k\ell>1$. This is the range of wavenumbers for which the acoustic wavelength approximately equals the bending wavelength.

a. For $\frac{1}{2}kw>>1$, $R_{rad} = S\rho c(k\ell)^{1/2}/3\sqrt{\pi}$.

b. For $\frac{1}{2}kw<1$, $R_{rad} = \frac{1}{4}S\rho ckw$.

4. $\frac{1}{2}k\ell<1$ with $\frac{1}{2}kw<1$.

$$R_{rad}(\text{odd}) = S\rho c(k/k_b)^2(wk/2\pi\ell k)\{1/[1-(k/k_b)^2] + (k_b/2k)\log(k_b+k)/(k_b-k)\}$$

and

$$R_{rad}(\text{even}) = S\rho c(k\ell kw/2\pi)\{1/[1-(k/k_b)^2] - (k_b/2k)\log(k_b+k)/(k_b-k)\}$$

The resulting expressions for $R_{rad}$ are approximate solutions of eq. (1). The various restrictions placed on wavenumber and hence wavelength, and the restrictions placed on the size of the panel in relation to the acoustic wavelength are simply those which will make possible approximate solutions of eq. (1). And while it should be noted that these expressions for $R_{rad}$ have considerable scientific value, the engineering usefulness is relatively limited.

## Flat Plates

In 1954, W. Westphal developed equations for the power radiated from flat plates under various conditions.[3] From these results for the power radiated from flat plates, the radiation resistance can be calculated by noting that[4]

$$R_{rad} = S\rho c\gamma, \tag{2}$$

where the radiation factor,[5] $\gamma = 2P/\rho cS\overline{v}^2$. Hence,

$$R_{rad} = 2P/\overline{v}^2.$$

Westphal's results are expressed in terms of power radiated per unit of area; therefore, for an infinitely large plate with

1. A homogeneous material whose thickness is small in relation to the bending wavelength
2. Small damping which can be expressed in terms of a complex modulus

3. Traveling, plane bending waves;

the power radiated is

$$P/S = \frac{1}{2}\rho c\bar{v}^2\gamma,$$

where the radiation factor is dependent on damping,

$$\gamma = \omega\beta_3/c(\beta_3^2 + \alpha_3^2).$$

The quantities $\beta_3,\alpha_3$ are the real and imaginary parts of the wavenumber in the direction perpendicular to the plate. They are related to $\beta_1,\alpha_1$, the real and imaginary parts of the wavenumber in the direction of bending wave propagation on the plate by

$$\alpha_3^2 = \alpha_1^2\beta_1^2/\beta_3^2$$

$$\beta_3^2 = \frac{1}{2}[(\omega^2/c^2)+\alpha_1^2-\beta_1^2] + \frac{1}{2}\sqrt{[(\omega^2/c^2)+\alpha_1^2-\beta_1^2]^2 + 4\alpha_1^2\beta_1^2},$$

where $\beta_1 = \sqrt{\omega\omega_g}/c$ and $\alpha_1\lambda = 0.115D$. From this, the radiation resistance results as

$$R_{rad} = \rho cS\omega\beta_3/c(\beta_3^2+\alpha_3^2).$$

A plot of $\gamma$ for various values of D follows in Figure 3.[3] This figure illustrates the behavior of $R_{rad}$ as a function of D.

For the undamped case $(\alpha_1=\alpha_2=0)$, $R_{rad}$ is

$$R_{rad} = \rho cS\omega/c\beta_3 = \rho cS/\sqrt{1-(\omega_g/\omega)} = \rho cS/\sqrt{1-(\lambda/\lambda_B)^2},$$

for $\lambda_B \geq \lambda$, and $R_{rad}=0$ for $\lambda_B<\lambda$.

An analysis of the radiation produced by means of standing waves in an infinite plate yields

$$R_{rad} = \rho cS/\sqrt{1 - (n_x\lambda/2a)^2 - (n_y\lambda/2b)^2}$$

for cases of the subdivision on the plate being larger than the acoustic wavelength at a given frequency. And $R_{rad}=0$ for cases of the subdivisions on the plate being smaller than the acoustic wavelength.

Westphal goes on to establish an effective radiation factor for the case of plate vibration produced by a band of excitation frequencies. This result, however, is most useful when used for experimental determination of the radiation factor and, hence, the $R_{rad}$. In fact, this result was used to obtain experimental values of the radiation factor in Westphal's work.

## Orthotropic Plates

The sound power radiated from infinite orthotropic plates has been studied,[6] and from this work information regarding the radiation resistance can be deduced.

The orthotropic plates studied are those plates that possess different bending strengths in different directions. This difference in bending strengths may be due to corrugations or ribs that run in one direction. Such characteristics may also result from a directionality in stiffness of the material (wood would be an example). Ribbed plates have also been investigated, but that case, which will be considered later, involved the ribs forming a grid or grill.[7]

The expressions developed for the sound power radiated from infinitely large orthotropic plates were derived from the differential equation of velocity. Such results are restricted by the following assumptions:

1. The flexural wavelengths in the plate must be greater than the distance between the inhomogeneties of the plate.
2. Ribs or corrugations run in one direction only.
3. Damping is small and can be considered constant, independent of direction.
4. The excitation is a point force.

From expressions for the radiated power,[6] by using the radiation factor, $\gamma$, due to K. Gosele,[5] eq. (2) can be used to obtain the radiation resistance. The results are expressions in three frequency ranges:

1. $k^4<(\omega^2 m/B_x)$, which means that the largest bending wavelengths in the plate are smaller than the lower critical wavelength in the surrounding medium.

$$R_{rad} \approx (32/\pi)(S\rho\omega\eta/cz_o)\sqrt{B_x B_y} \qquad (3)$$

2. $(\omega^2 m/B_x)<k^4<(\omega^2 m/B_y)$. This range of frequency includes bending wavelengths in the plate which are greater than the lower critical wavelength in the surrounding medium in one direction, but less than the upper critical wavelength in the surrounding medium in the other direction.

$$R_{rad} \approx S\rho c(1/\pi^2)\sqrt{f_{g1}/f_{g2}}[\ln(4f/f_{g1})]^2$$

3. $k^4 > (\omega^2 m/B_y)$. In this range all wavelengths of the bending waves in the plate are greater than the upper critical wavelength in the surrounding medium.

$$R_{rad} \approx S\rho c.$$

Thus, as compared to simple plates that have one critical frequency, orthotropic plates have two critical frequency values. One will be produced by the bending strength, $B_x$, in the largest stiffness direction and the other will be produced by the bending strength, $B_y$, in the smallest stiffness direction. And obviously if $B_x = B_y$, the plate is not orthotropic. In such cases, eq. (3) reduces to a form comparable to previous results for flat plates.

## Ribbed Panels

### Single-mode Vibration

The radiation resistance of ribbed panels has been investigated.[7] The study began on the same fundamental footing as the work done on beams (see the previous section on beams). The general equation for the radiation resistance (see eq. (1)) serves as the "master" equation from which specific results are developed. Also it should be noted that the assumptions and simplifcations made in the determination of $R_{rad}$ for the beam still apply.

The following results are valid for a single-mode vibration of a finite, simply supported, baffled panel. These results are presented as being the dominant terms in the evaluation of eq. (1) in each frequency range.

1. $(k_b/k)<1$ (above coincidence)

$$R_{rad} \approx S\rho c[1-(k_b/k)^2]^{-1/2}$$

2. $k_b \approx k$ (in the neighborhood of coincidence)

$$R_{rad} \approx (S\rho c/3\sqrt{\pi})[(\ell k_b^2/k_{bx})^{1/2} + (hk_b^2/k_{by})^{1/2}]$$

3. $(k_b/k)>1$ (below coincidence), with

a. $(k_{by}/k)>1$ and $(k_{bx}/k)<1$,

$$R_{rad}^{x} = \frac{S\rho c(k/k_{by})^2}{kh} \frac{1 + [(k_b^2-k^2)/k_{by}^2]}{[(k_b^2-k^2)/k_{by}^2]^{3/2}}$$

b. $(k_{bx}/k)>1$ and $(k_{by}/k)<1$,

$$R_{rad}^{y} = \frac{S\rho c(k/k_{bx})^2}{k\ell} \frac{1 + [(k_b^2-k^2)/k_{bx}^2]}{[(k_b^2-k^2)/k_{bx}^2]^{3/2}}$$

4. $(k_b/k)$, $(k_{bx}/k)$, $(k_{by}/k)<<1$ (well below coincidence) with $\frac{1}{2}k\ell$, $\frac{1}{2}kh>1$,

$$R_{rad} = (8\rho c/\pi)k^2/(k_{bx}k_{by})^2$$

5. $\frac{1}{2}k\ell$, $\frac{1}{2}kh<<1$

a. For odd-odd modes of vibration,

$$R_{rad}^{oo} \approx [(32\rho c/\pi)k^2/(k_{bx}k_{by})^2]\{1 + 0[(\tfrac{1}{2}k\ell)^2+(\tfrac{1}{2}kh)^2]\}$$

b. For even in the x-direction and odd in the y-direction,

$$R_{rad}^{eo} \approx (32\rho c/3\pi)[k^2/(k_{bx}k_{by})^2](\tfrac{1}{2}k\ell)^2[1 + 0(\tfrac{1}{2}kh)^2]$$

c. For even in the y-direction and odd in the x-direction,

$$R_{rad}^{oe} \approx (32\rho c/3\pi)[k^2/(k_{bx}k_{by})^2](\tfrac{1}{2}kh)^2[1 + 0(\tfrac{1}{2}kh)^2]$$

d. For even-even modes of vibration,

$$R_{rad}^{ee} \approx (32\rho c/15\pi)[k^2/(k_{bx}k_{by})^2](\tfrac{1}{2}k\ell)^2(\tfrac{1}{2}kh)^2$$

6. $\frac{1}{2}kh<<1$ and $\frac{1}{2}k\ell>1$, which corresponds to long, narrow panels and implies that $(k_{by}/k)>>1$.

a. For odd modes in the y-direction,

$$R_{rad}^{o} \approx \begin{cases} 4\rho ck\ell/k_{by}^2, & (k_{bx}/k)<1 \\ (16\rho c/\pi)k^2/(k_{by}k_{bx})^2, & (k_{bx}/k)>1 \end{cases}$$

b. For even modes in the y-direction,

$$R_{rad}^{e} \approx \begin{cases} (4\rho ck\ell/3k_{by}^2)(\tfrac{1}{2}kh)^2, & (k_{bx}/k)<1 \\ (16\rho c/3\pi k_{bx}^2k_{by}^2)(\tfrac{1}{2}kh)^2, & (k_{bx}/k)>>1 \end{cases}$$

## Multi-mode Vibration

The mathematical expressions previously given for the single-mode radiation resistance of panels have limited value in the engineering world, since multi-modal vibration occurs most frequently. More general equations for the prediction of the radiation resistance of panels whose vibration is reverberant follow.[7]

$$R_{rad} = S\rho c[(\lambda\lambda\rho/S)g_1(f/f_g) + (P_r\lambda p/S)g_2(f/f_g)]$$

in the range, $f<f_g$ with $\frac{1}{2}k\ell$, $\frac{1}{2}kh>1$.

$$R_{rad} = S\rho c[(\ell/\lambda_p)^{1/2} + (h/\lambda_p)^{1/2}],$$

at the point, $f=f_g$, for $\frac{1}{2}k\ell$, $\frac{1}{2}kh>1$, and

$$R_{rad} = S\rho c[1 - (f/f_g)]^{-1/2},$$

for $f>f_g$ with $\frac{1}{2}k\ell$, $\frac{1}{2}kh>1$, and where

$$g_1(f/f_g) = \begin{cases} (4/\pi^4)(1-2\alpha^2)/\alpha(1-\alpha^2)^{1/2}, & f<\frac{1}{2}f_g \\ 0 & , \quad f>\frac{1}{2}f_g \end{cases}$$

$$g_2(f/f_g) = \frac{1}{(2\pi)^2}\,\frac{(1-\alpha^2)\ln[(1+\alpha)/(1-\alpha)] + 2\alpha}{(1-\alpha^2)^{3/2}},$$

$P_r = 2(\ell+h)$, and $\alpha = (f/f_g)^{1/2}$.

For the range, $\frac{1}{2}k\ell$, $\frac{1}{2}kh<<1$,

$$R_{rad} = S\rho c(4/\pi^4)(P_r\lambda_p/S)(f/f_g)^{1/2}.$$

In the determination of these expressions for radiation resistance, the contribution from even-even, odd-even, and even-odd modes was neglected as being small compared to the contribution due to odd-odd modes. It was also assumed that $k_b\ell$, $k_bh>>\pi$.

Below the critical frequency, $f<f_g$, it has been noted that the radiation resistance is directly proportional to the perimeter of the panel.[7] Thus the conclusion is drawn that a ribbed panel can be treated in the same fashion as an unribbed panel. All account can be made for the effect of the ribs on the radiation output by redefining the perimeter. Hence, for a ribbed panel, the radiation resistance will be

increased by an amount

$$(P_r^{ribs} + P_r^{panel})/P_r^{panel},$$

where $P_r^{ribs}$ is twice the total length of the ribs and $P_r^{panel}$ is the perimeter of the panel. However, when this technique is used to estimate the radiation resistance of a ribbed panel, the spacing between the ribs must now be restricted with respect to the acoustic wavelength in the same way as the dimensions of the entire unribbed panel. This means that distance between adjacent ribs must be large when compared to the acoustic wavelength. The factor by which this distance must be larger than the acoustic wavelength in order for these results to be reasonably accurate has not been discussed, but this is under current consideration.

## Cylindrical Shells

Qualitative consideration of the "radiation properties" of cylindrical shells has been carried out,[8] but currently no useful engineering results exist for predicting the radiation resistance proper for multimodal vibration. This is an area toward which future efforts of this research activity will be directed.

## Conclusions

The true value of being able to determine $R_{rad}$ analytically should be stated. From the equation,[7]

$$S_a(\omega)/S_p(\omega) = \Gamma(\omega)\mu(\omega),$$

where $\Gamma(\omega) = [2\pi^2 n_p(\omega)/M](c/\rho)$ and $\mu(\omega) = R_{rad}/(R_{rad}+R_{mech})$, the relationship between the motion of the structure and the motion of the acoustic medium is established. Now knowing either $S_a$ or $S_p$, the other can be determined provided $\Gamma(\omega)$ and $\mu(\omega)$ can be found.

$\Gamma(\omega)$ depends on the variables, $c$, $\rho$, $M$, and $n_p(\omega)$; $c$, $\rho$, and $M$ are easily determined; and $n_p(\omega)$, the modal density of the structure, is known from flat plates[9] and cylinders[9,10] in analytical form. Hence, only the resistance ratio, $\mu(\omega)$ remains. And if $R_{rad}$ can be computed analytically, only $R_{mech}$ needs to be determined experimentally. $R_{mech}$ (in truth, $R_{rad}+R_{mech}$) can be determined experimentally by measuring the reverberation time, $T_S$, of the structure, which is relatively simple to do.

If $R_{rad}$ cannot be evaluated analytically, then both $S_a$ and $S_p$ must be known. This will require the experimental determination of the spectral density, $S_a$ or $S_p$, and the reverberation time. Experimental determination of the spectral densities is neither as simple nor as inexpensive to perform as is determination of the reverberation time. Therefore, the ability to predict $R_{rad}$ analytically is of much value in both time and money.

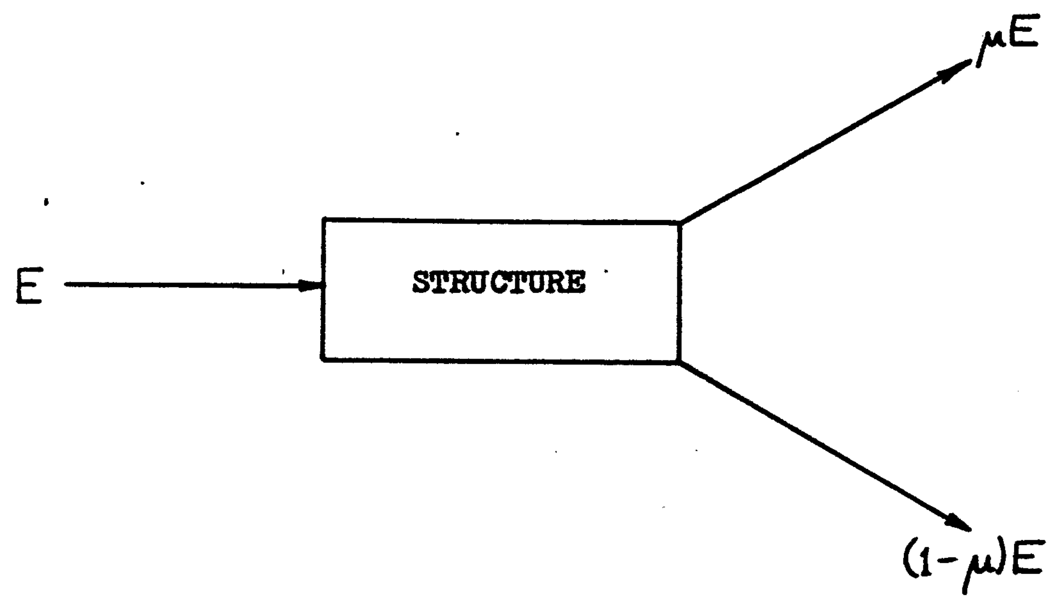


FIG. 1: SCHEMATIC ILLUSTRATION OF ENERGY FLOW

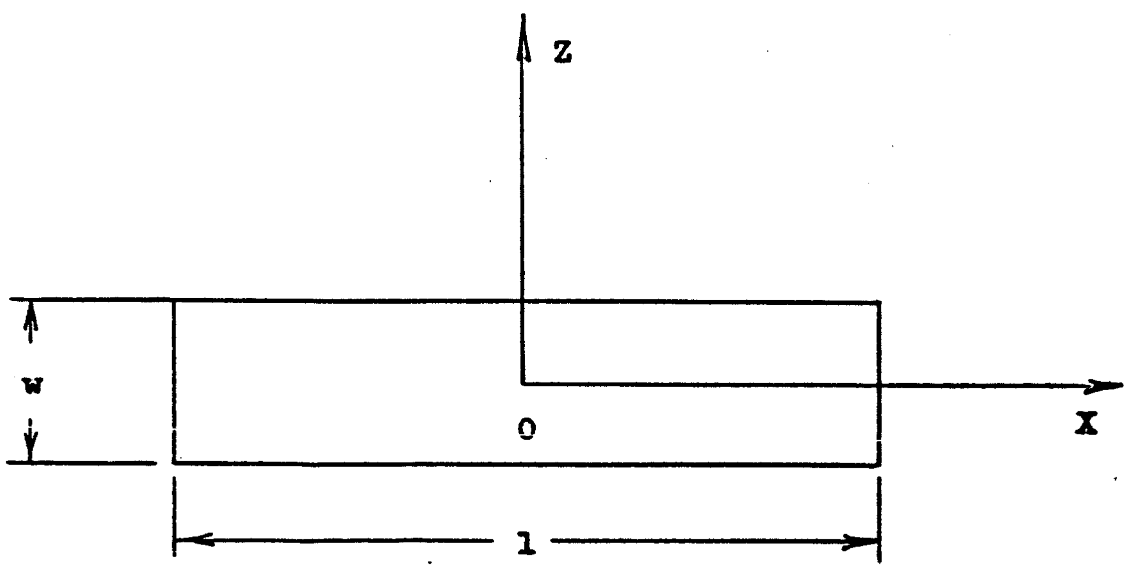


FIG. 2: SIMPLY SUPPORTED BEAM WITH DIMENSIONS
AND REFERENCE SHOWN

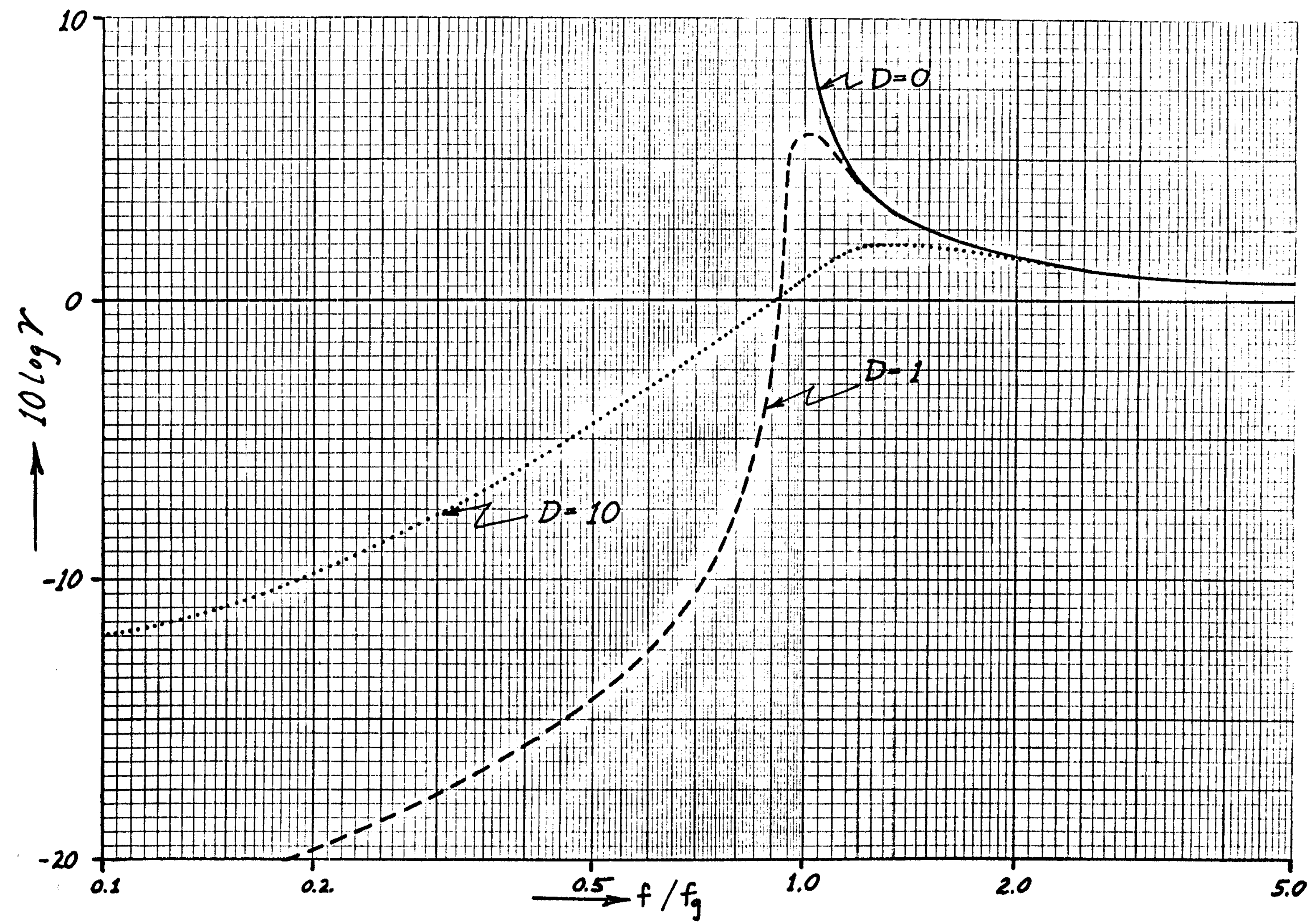


FIG. 3: RADIATION FACTOR VS FREQUENCY RATIO FOR VARIOUS VALUES OF D

## Notation

| | |
|---|---|
| $a$ | = characteristic length in the x-direction of an infinite plate subject to standing wave vibration. |
| $b$ | = characteristic length in the y-direction of an infinite plate subject to standing wave vibration. |
| $B_x$ | = orthotropic plate bending stiffness in the x-direction. |
| $B_y$ | = orthotropic plate bending stiffness in the y-direction. |
| $c$ | = speed of sound in the acoustic medium. |
| $c_o$ | = damping coefficient. |
| $c_c$ | = critical damping coefficient. |
| $D$ | = damping per wavelength. |
| $E$ | = energy. |
| $f$ | = frequency (cps). |
| $f_g$ | = critical frequency of the plate. |
| $f_{g1}$ | = lower critical frequency. |
| $f_{g2}$ | = upper critical frequency. |
| $h$ | = width of the plate. |
| $k$ | = mean acoustic wave number in the frequency band of excitation. |
| $k_b$ | = $\pi n/\ell$, mean wavenumber of the bending vibrations in the structure. |
| $k_{bx}$ | = mean wavenumber in the x-direction. |
| $k_{by}$ | = mean wave number in the y-direction. |
| $\ell$ | = length of the beam or plate. |
| $m$ | = mass per unit of area. |
| $M$ | = lumped mass of the system |

$n_p$ = modal density of the structure.

$n_R$ = modal density of the chamber in which the plate is tested.

$n_x$ = 0,1,2,3,....

$n_y$ = 0,1,2,3,....

$P$ = power radiated from the structure.

$R$ = resistance.

$R_{mech}$ = mechanical or internal resistance.

$R_{rad}$ = radiation resistance.

$S$ = surface area of the structure.

$S_a$ = spectral density of the acceleration of the structure.

$S_p$ = spectral density of the acoustic pressure in the medium surrounding the structure.

$T_s$ = reverberation time of the system.

$T_R$ = reverberation time of the test chamber.

$\overline{v}^2$ = mean square velocity of response on the structure.

$v_o^2$ = velocity of response at the point of force application.

$w$ = width of the beam.

$\underline{x}$ = position vector to a point on the surface of the structure.

$z_o$ = impedance at the point of force application.

$\alpha(\ )$ = imaginary part of the complex wavenumber in the ( )-direction.

$\beta(\ )$ = real part of the complex wavenumber in the ( )-direction.

$\gamma$ = radiation factor.

$\eta$ = loss factor.

$\eta_{mech}$ = mechanical or internal loss factor.

$\eta_{rad}$ = radiation loss factor.

$\lambda$ = acoustic wavelength.

$\lambda_B$ = bending wavelength.

$\lambda_p$ = coincidence wavelength of the plate.

$\mathfrak{R}$ = resistance ratio.

$\rho$ = ambient density of the acoustic medium.

$\phi(\underline{x}_1, \underline{x}_2)$ = correlation of the structural vibration field.

$\psi(\underline{x}_1, \underline{x}_2)$ = correlation of the acoustic field.

$\omega$ = circular frequency (radian/sec)

$\omega_o$ = resonant frequency of a single mode.

$\omega_g$ = critical circular frequency.

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# APPENDIX B

## LOW-FREQUENCY NOISE REDUCTION

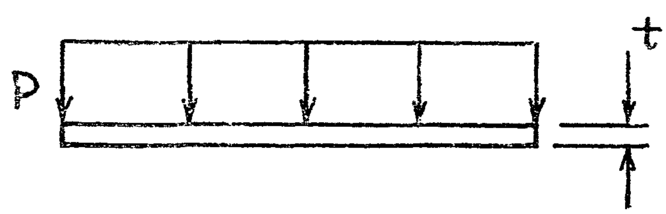
J. Ronald Bailey

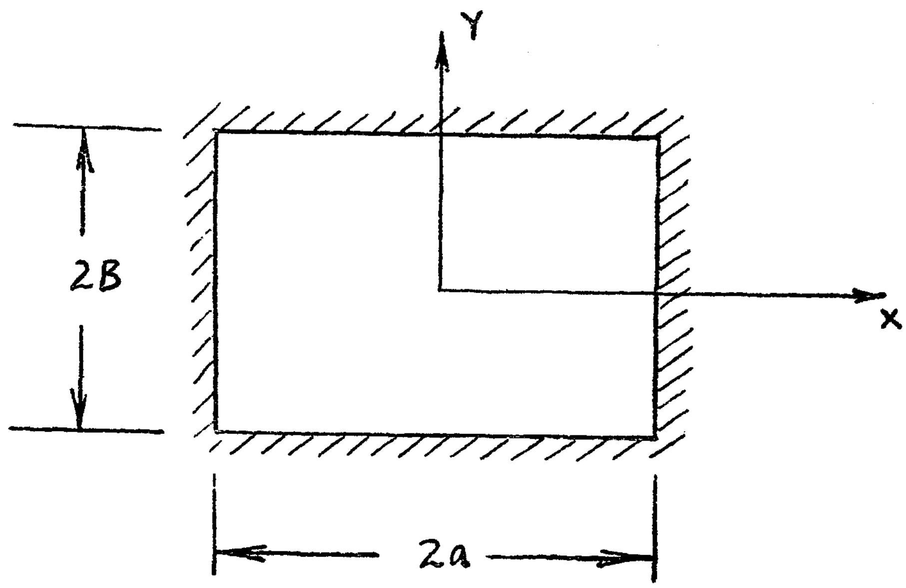
The interaction of acoustical and structural vibrations is important in the field of noise reduction. In this paper, the noise reduction in a small pentagonal enclosure with two flexible walls is studied in the low-frequency range where both the panel and the interior volumes are stiffness controlled. This is the first phase of a statistical approach to noise reduction in all frequency ranges.

Consider the enclosure shown in Figure 1. The "roof" panels are flexible and the enclosure is rigid. Exposure to an external sound pressure, P, will result in an internal pressure, $P_b$, given by[1]

$$P_b = X/C_b, \tag{1}$$

where X is the volume displacement due to P and $C_b$ is the acoustic compliance; $C_b = V_b/\rho c_a^2$. The volume displacement can be calculated by considering a uniformly loaded flat rectangular plate with clamped edges, as illustrated below.






The displacement is given by[2]

$$w = P(a^2-x^2)^2(b^2-y^2)^2/24D(a^4+b^4).$$

The volume displacement is thus

$$x = \int_{-a}^{a}\int_{-b}^{b} w(x,y)\,dxdy = 4\int_{0}^{a}\int_{0}^{b} w(x,y)\,dxdy.$$

Substituting and integrating gives

$$x = 32Pa^5b^5/675D(a^4+b^4).$$

Let $b/a = r$. Then,

$$x = 32Pa^6r^5/675D(1+r^4).$$

Since there are two flexible panels, the total volume displacement is

$$X = 2x = 64Pa^6r^5/675D(1+r^4).$$

Let $64r^4/675(1+r^4) = F(r)$. Then

$$X = Pa^6 rF(r)/D. \qquad (2)$$

F(r) versus r is plotted in Figure 2.

The interior volume is

$$V_b = (hcb/2) + c^2 b, \qquad (3)$$

but $c = 2\sqrt{a^2-h^2}$, $r = b/a$, and letting $s = h/a$, equation (3) can be written,

$$V_b/ra^3 = [s\sqrt{1-s^2} + 4(1-s^2)] = F(s). \qquad (4)$$

Figure 3 is a plot of F(s) versus s. Now the volume compliance can be written,

$$C_b = V_b/\rho c_a^2 = ra^3 F(s)/\rho c_a^2 \qquad (5)$$

Substituting (2) and (5) into (1) gives

$$P_b = Pa^3 \rho c_a^2 F(r)/DF(s)$$

or

$$P/P_b = DF(s)/a^3 \rho c_a^2 F(r),$$

where $D = \frac{1}{12} \rho_p c_1^2 t^3$ and $c_1 = \sqrt{E/\rho(1-\sigma^2)}$.

Let $K = t/a$,

$$NR = 20 \log[EF(s)/12(1-\sigma^2)\rho c_a^2 F(r) K^3], \qquad (6)$$

where E is Young's modulus; F(s) is the height ratio function (see Figure 3); $\sigma$ is Poisson's ratio; $\rho$ is the density of air; $c_a$ is the velocity of sound in air; F(r) is the aspect ratio function (see Figure 2); and K (thickness ratio) = t/a.

Figure 4 is a plot of noise reduction for a box with dimensions a, h = $a/\sqrt{2}$, c = $\sqrt{2}$ a, and b = $2\sqrt{2}$ a.

## References


1. R. H. Lyon, "Noise Reduction of Rectangular Enclosures with One Flexible Wall," JASA 35, 11, 1791-1797 (1963).

2. A. E. H. Love, The Mathematical Theory of Elasticity, Dover Publications, 1944.

3. A. J. Pretlove, "Free Vibrations of a Rectangular Panel Backed by a Closed Rectangular Cavity," J. Sound Vibrations 2, 3, 197-209.

4. Albert London, "Transmission of Reverberant Sound Through Single Walls," J. Research Natl. Bur. Standards, RP 1998, 42, 605-615 (1949).

5. Leo L. Beranck, Noise Reduction, McGraw-Hill Book Company, Inc., 1960.

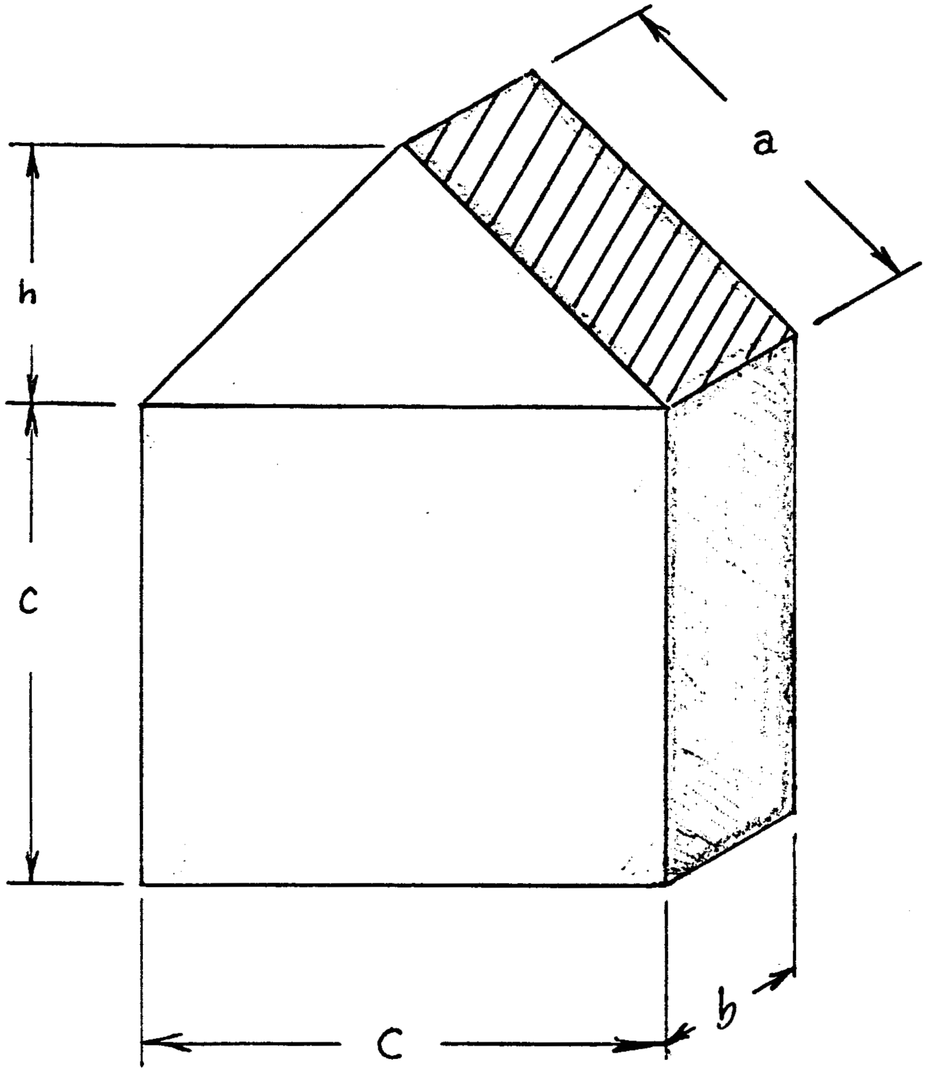



FIG. 1: PENTAGONAL ENCLOSURE WITH TWO FLEXIBLE WALLS

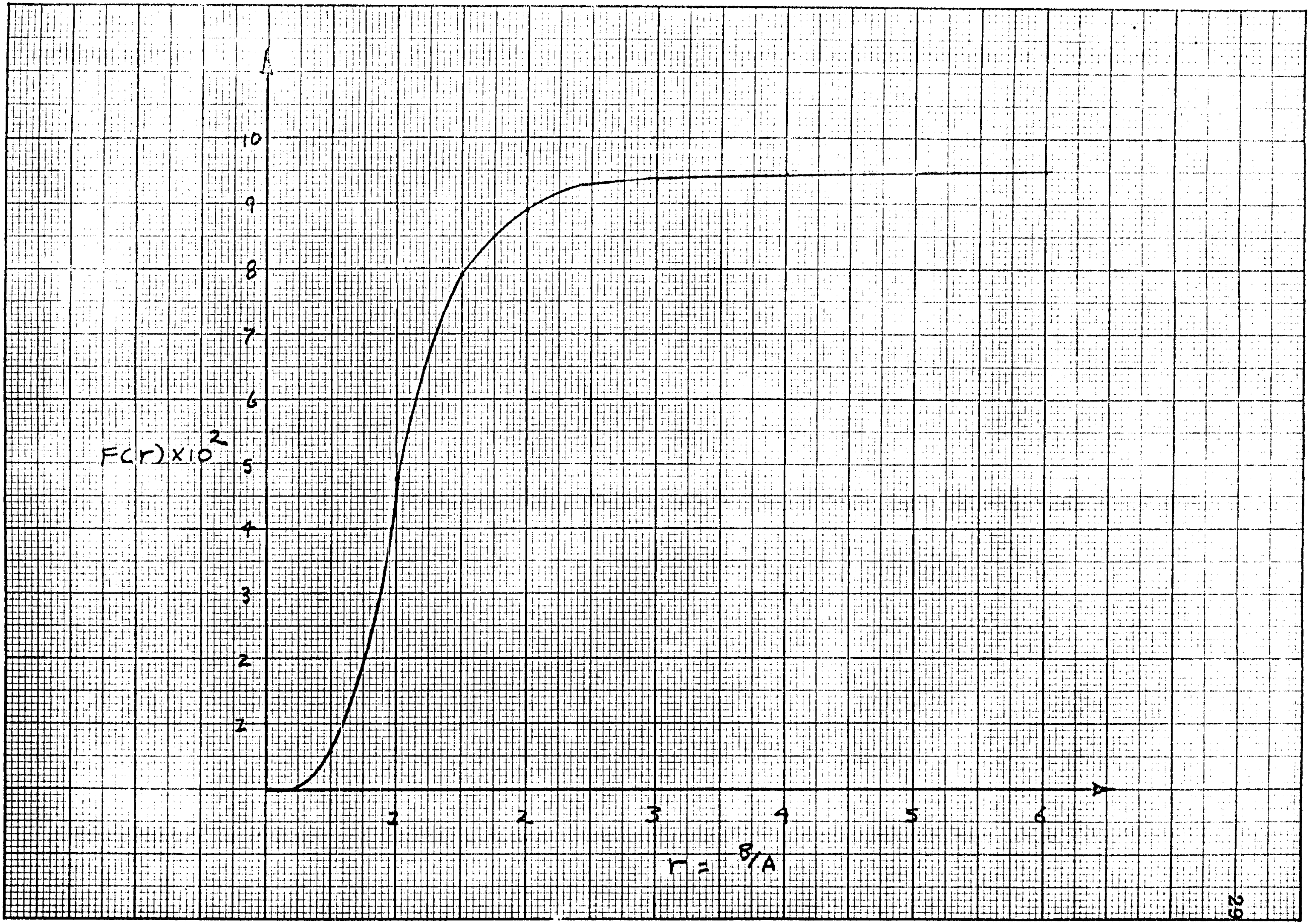



FIG. 2: F(r) VERSUS r FOR A PENTAGONAL ENCLOSURE

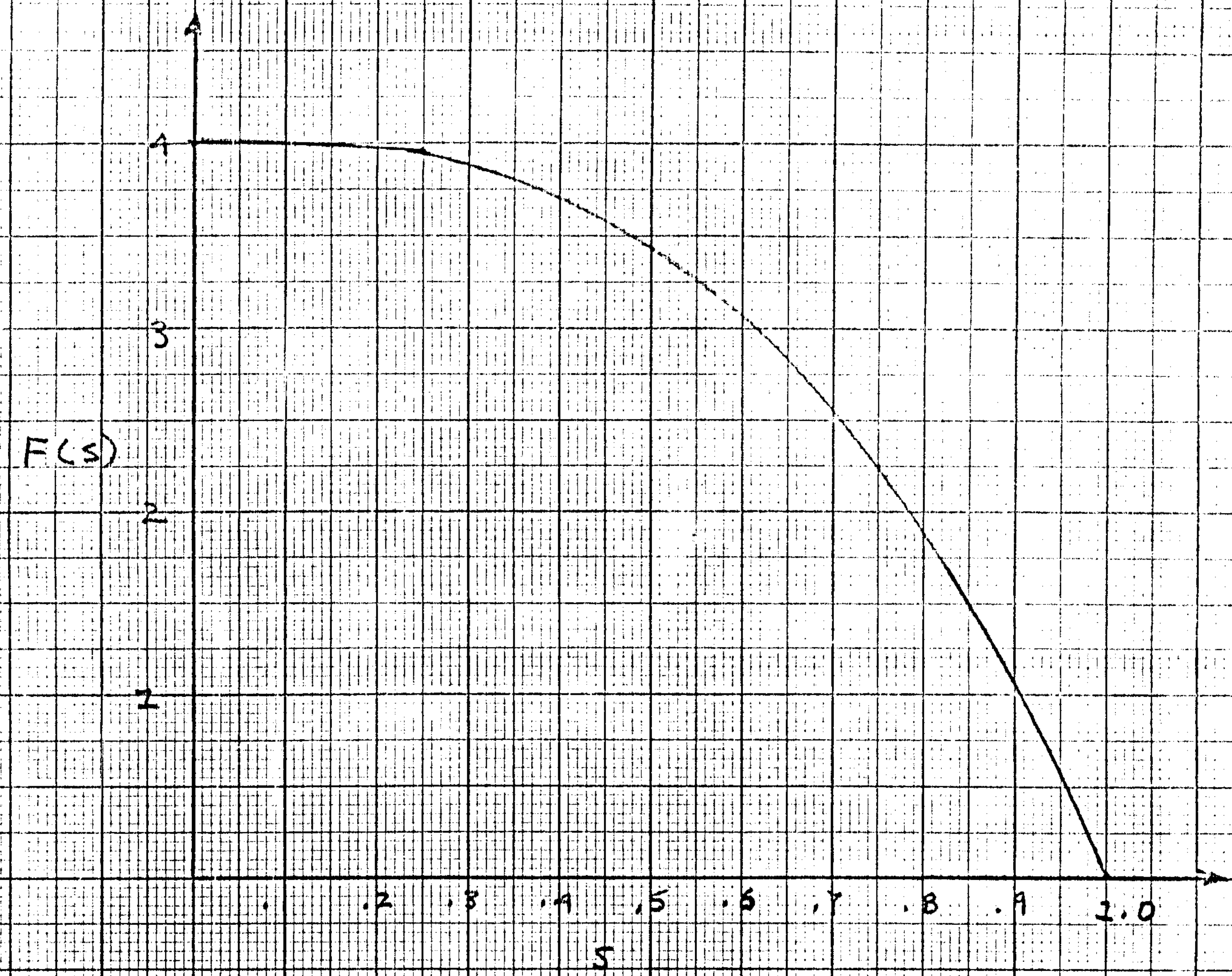



FIG. 3: F(s) VERSUS s FOR A PENTAGONAL ENCLOSURE

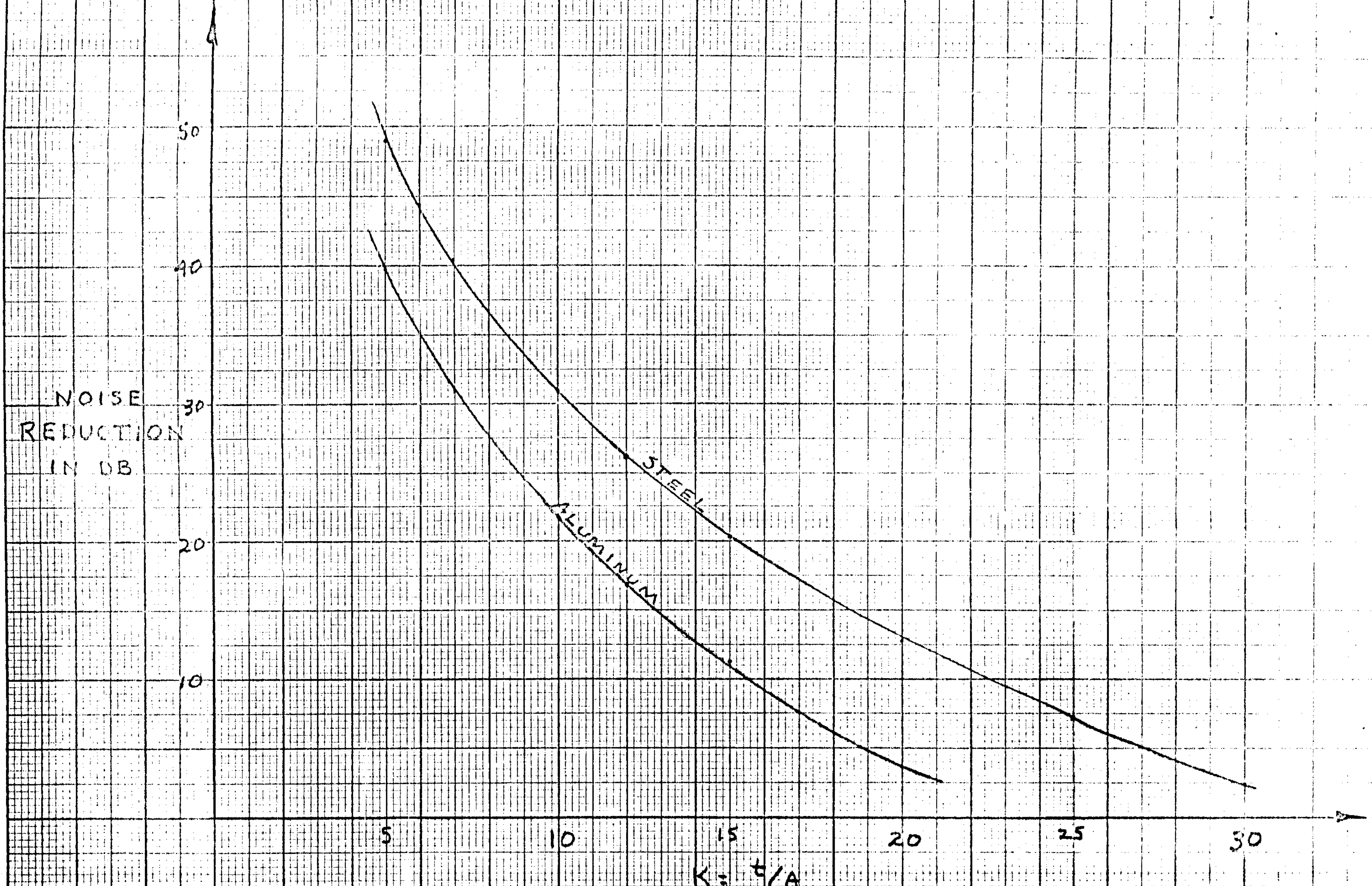



FIG. 4: NOISE REDUCTION FOR THE BOX GIVEN IN THE TEXT

# APPENDIX C

## MODAL DENSITY OF THIN CYLINDRICAL SHELLS

David K. Miller

The work that has been done thus far on modal densities of thin cylindrical shells, as presented herein, is based on V. V. Bolotin's paper, "On the Density of the Distribution of Natural Frequencies of Thin Elastic Shells,"[1] and Manfred Heckl's paper, "Vibrations of Point-driven Cylindrical Shells."[2] Both writers presented a derivation of expressions for the modal density of thin cylindrical shells. Heckl dealt with the modal density of a finite simply supported cylindrical shell and obtained the expressions:

For $\nu > 0$,

$$\Delta N/\Delta\nu = \ell/4a\beta = \sqrt{3}(\ell/2h),$$

and for $\nu < 0$,

$$\Delta N/\Delta\nu = [\tfrac{1}{2}\pi + \text{arc sin}(2\ -1)](\ell/4\pi a\beta),$$

where $\ell$ is the cylinder length; a, the cylinder radius; h, the thickness; $\beta = h/2\sqrt{3}\ a$; and $\nu$ is the dimensionless frequency given by

$$\nu = \omega a/c_2 = \omega a\sqrt{\rho/E},$$

where $\omega$ is the exciting frequency; $\rho$, the density; and E, the modulus of elasticity of the cylinder material.

Bolotin, on the other hand, derived a general expression for any thin elastic shell, which he then applied to the specific case of a thin cylindrical shell. Bolotin's expressions for the modal density are:

$$dN(\Omega)/d\Omega \approx (a_1 a_2/4\pi)(\rho h/D)^{1/2} H_1[(\Omega_R/\Omega),\mathcal{X}],$$

where for the specific case of a cylinder, $(\Omega_R/\Omega) = \alpha$ and $\mathcal{X} = 0$. Hence, for $\alpha<1$,

$$H_1(\alpha,0) = (2/\pi\sqrt{1+\alpha})K[\sqrt{2\alpha/(1+\alpha)}],$$

and for $\alpha>1$,

$$H_1(\alpha,0) = (r_2/\pi\sqrt{\alpha})K[\sqrt{(1+\alpha)/2\alpha}],$$

where $a_1$ and $a_2$ are the dimensions of the rectangular shell surface; $\rho$, the density; h, the thickness; D, the plate stiffness; $\alpha$, a dimensionless frequency parameter; and K, the eleptic integral of the first kind.

It was first felt necessary to determine to what extent the equations obtained by Heckl and Bolotin were comparable. This was accomplished by converting Bolotin's notation to that of Heckl, and noting that $\nu = \frac{1}{2}$, $a_1 = \ell$, and $a_2 = \pi a$, which reduces Bolotin's general expression to:

$$\Delta N/\Delta\nu = \sqrt{3}(\ell/2k)(1-\mu^2)^{1/2} H_1(\tfrac{1}{\nu},0).$$

The approximate values for $H_1(\frac{1}{\nu},0)$ were determined by means of Fig. 2 of Bolotin's paper, which is a plot of $H_1$ versus $\nu$ for $\mathcal{X}=0$. In this way it was possible to plot the results of both Heckl and Bolotin for a range of $\nu$ values (see Figure 1). It may be noted that for $\nu>1$, Heckl's modal density is $\sqrt{3}(\ell/2h)$, while Bolotin's converges to $\sqrt{3}(\ell/2h)(1-\mu^2)^{1/2}$. Hence, for frequencies above the ring frequency ($\nu>1$) the results of both papers appear to be in agreement. However, below the ring frequency there appears to be considerable difference between the two equations. Hence, this will be the range of primary interest in further work.

As a next step, an analytical review of Heckl's derivation is being undertaken to determine, if possible, the reason for the discrepancy in the modal density expression for frequencies below the ring frequency. This analysis is not as yet complete; however, in the analysis it was noted that Heckl introduced an approximate frequency equation in order to obtain a perfect square term for $\nu^2$. The approximation was as follows:

$$\nu^2 = [\sigma^2(n^2+\sigma^2)^{-1} + \beta(k^2+\sigma^2)]^2,$$

where $\beta = (h/2)/\sqrt{3}\ a$ and $\sigma = m\pi a/\ell$. It is evident from Heckl's derivation that this approximation is based on the assumptions that $\mu\approx 0$, and that the terms, $-2n^2\beta - 2\sigma^2\beta + \beta^2$, may be neglected in the final frequency equation. To get some idea of the meaning of this assumption, a set of cylinder dimensions was chosen ($a=2.25''$, $h=0.062''$, and $\ell=36''$), and with this data it was possible to plot Heckl's approximate value for $\nu^2$, as well as a plot of the approximate value plus the terms that were neglected for various values of n ($n = \frac{1}{2}$ number of nodes in the circumferential direction) using m as a parameter. Where $k=m\pi/\ell$, $m=1,2,3,\cdots$ is the wavenumber in the axial direction (see Figure 2). Heckl stated that the approximation is good except where $6^2 \approx (n^2+\sigma^2)^2\beta$, and that this range is fairly small. It was found that the above equality is nearly satisfied, for the range of values chosen, in the vicinity of n=5. It may be noted that although this range is fairly small it seems to have its greatest effect for values of $\nu<1$, whereas for $\nu>1$ the values of the approximation are seen to converge with the actual values. Heckl also stated that his approximation may lead to frequencies which are as much as 40% too high in this range, and this is nearly the case for the m=14 curves shown in Figure 2. Since this error seems to be fairly important below the ring frequency, it may be concluded that, since the error in the frequency is on the high side, the expression which Heckl derived for the modal density is somewhat lower than would actually be the case for frequencies lower than the ring frequency. The exact effect of this approximation on Heckl's modal density expression has not been determined, but it would seem that it has lowered the expression somewhat, although work is still in progress to clarify this matter. An analysis of this sort on Bolotin's paper also will be made at some future date.

Also in progress at this time is an evaluation of modal density predictions presented by Heckl and Bolotin by experimental means. Thus far the necessary test equipment has been obtained and set up (see Figure 3). It is desired to excite the test cylinder with a constant input force, and to measure the output acceleration at various points on the cylinder. Using the level recorder it is hoped that it will be possible to count the resonate peaks up to the vicinity of the ring frequency, and in this way determine the modal density experimentally.

A stainless steel cylinder also has been acquired for the initial test work. The dimensions of the cylinder are as follows: length, 36"; thickness, 0.065"; outside diameter, 4.5". This cylinder size was chosen because it has a ring frequency of about 13,000 Hz, which is within the range of the oscillator, and because the cylinder size is in the range between the two cylinders used by Heckl in his experimental work.

Work has been done to evaluate different mounting methods for the pick-up accelerometer. Since it is not desirable to drill holes in the cylinder to stud mount the accelerometer at the several locations to be used, various types of double-sided tape, as well as the wax supplied with the accelerometer, have been investigated with respect to their response as compared with that of a rigid mount. It has been found that on a flat surface the regular double-sided Scotch tape is a very good mounting method, as is the wax. Both methods provide an accelerometer response curve that is quite close to the stud mounted curve. However, due to the curvature of the cylinder, it appears that the wax mounting offers the best solution; at least this will be the method tried initially.

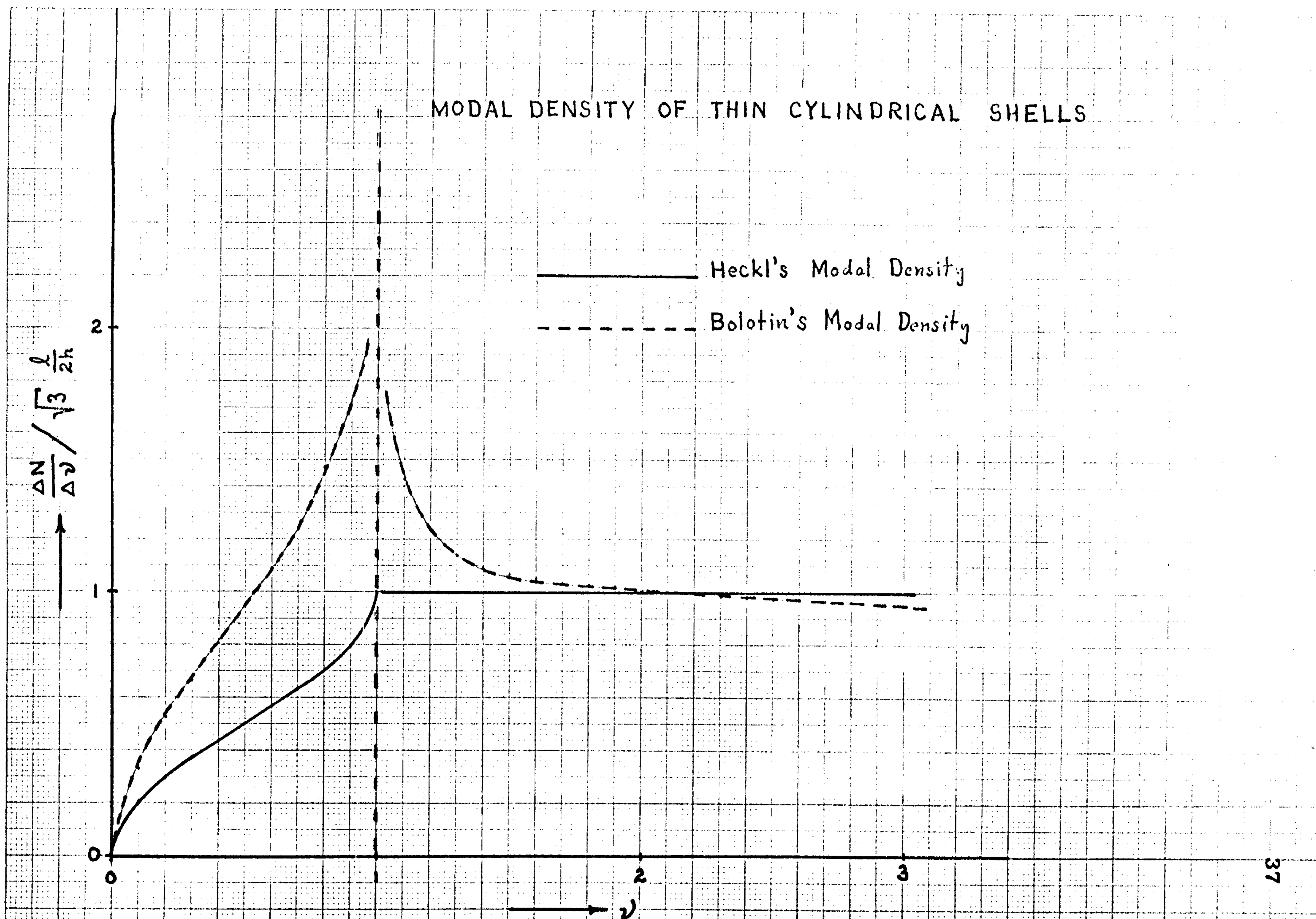

MODAL DENSITY OF THIN CYLINDRICAL SHELLS
Heckl's Modal Density
Bolotin's Modal Density
$\frac{\Delta N}{\Delta \nu} / \sqrt{3}\, \frac{\ell}{2h}$
$\nu$
0
1
2
3

Figure 1.

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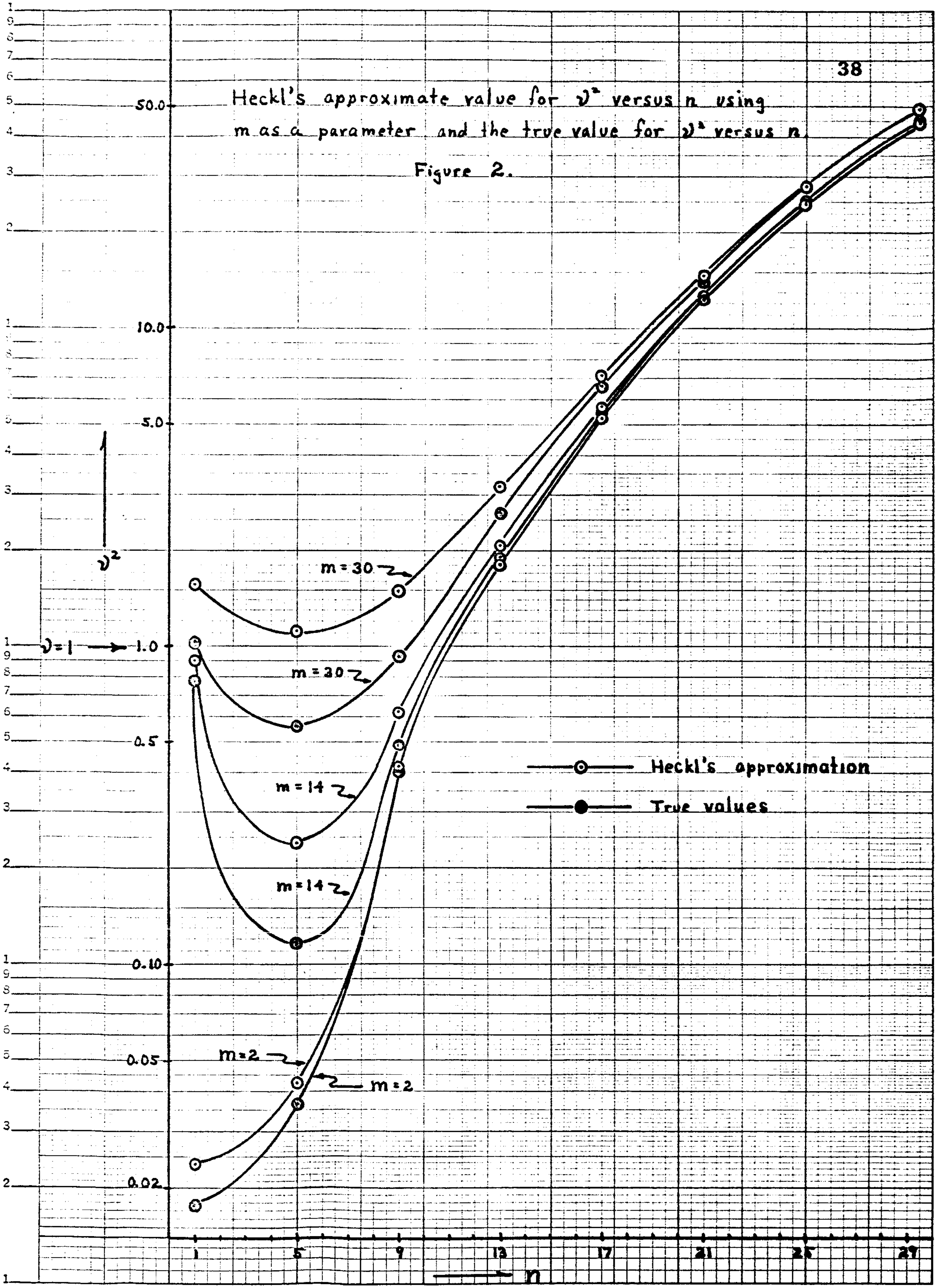


Heckl's approximate value for $\nu^2$ versus n using m as a parameter and the true value for $\nu^2$ versus n.

Figure 2.

Semi-Logarithmic
3 Cycles x 10 to the inch

## References

1. V. V. Bolotin, "On the Density of the Distribution of Natural Frequencies of Thin Elastic Shells," PMM 27, 538-543 (1963).

2. M. Heckl, "Vibrations of Point-driven Cylindrical Shells," J. Acoust. Soc. Am. 34, 1553-1557 (1962).